(2)

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
29 August 1979

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
THE TWO-BODY PROBLEM OF CLASSICAL ELECTRODYNAMICS

5. FUNDING NUMBERS

6. AUTHOR(S)
Rodney D. Driver

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Rhode Island
Kingston, RI 02881
Department of Mathematics

8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR.TR. 89-1411

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR
BLDG 410
BAFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR 77-3397

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Various competing mathematical models are used in classical electrodynamics. To test, and possibly eliminate, some of these models one can apply them to the two-body problem and see whether reasonable results are obtained. In one model it is assumed that each particle is influenced by both the past and future behavior of the other. The special case of two electrons moving symmetrically in one dimension was considered; and it was found that this curious model does make sense mathematically provided the two electrons never get too close together. Further studies under this grant led to a simple method for analyzing the asymptotic behavior of solutions of certain linear delay differential equations. This is useful in a one-body problem of electrodynamics with "radiation reaction", in problems of control theory with time lags, in the "telegraph equation", and other applications. Current work in progress is aimed at understanding the simplest n-body problem of electrodynamics with interactions occurring only through retarded fields.

14. SUBJECT TERMS

15. NUMBER OF PAGES
8

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
unclassified	unclassified		

NSN 7540-01-280-5500

Standard Form 298 (890104 Draft)
Prescribed by ANSI Std. 239-18
298-01

THE TWO-BODY PROBLEM OF CLASSICAL ELECTRODYNAMICS

Final Technical Report for AFOSR Grant 77-3397 and 77-3397A

Period covered: 1 July 1977 - 30 June 1979

Investigator: Rodney D. Driver
Professor of Mathematics
University of Rhode Island
Kingston, RI 02881

AFOSR-TR- 89-1411

Contents

I. RESEARCH OBJECTIVES

The research under this grant sought a better understanding of some of the various competing mathematical models for the interaction of charged particles.

In "classical" (as distinguished from quantum mechanical) electrodynamics three different types of models are in use:

A. In the simplest case each particle is influenced solely by the retarded fields produced by the other particles.

B. Another model has each particle moving under the influence of the retarded fields from the other particles plus its own "radiation reaction" field. Within this category, various different formulations for radiation reaction have been proposed.

C. The third alternative theory does not include any explicit radiation reaction terms. Instead it assumes that each particle is influenced by both retarded and advanced fields from the others.

Perhaps the simplest nontrivial problem to which one could apply each of these mathematical theories is that of studying the motion of just two charged particles assumed alone in space. An examination of questions of existence and uniqueness and the nature of solutions may show that one or more of the competing models indicated above is mathematically inconsistent or leads to unacceptable conclusions.

For any model which survives the two-body-problem test, one would naturally want to consider the three- and n-body problems.

II. STATUS OF THE RESEARCH EFFORT

Results known before the start of this grant (AFOSR 77-3397) included the following:

A. For the two-body problem with retarded interactions only, it had been found that in one space dimension one obtains well-posed mathematical problems leading to the expected types of solutions [4], [11], [19]. These results had been partially extended to motion in three dimensions [5], [8].

B. The two-body problem with retarded interactions plus the radiation reaction terms of Dirac [3] in one dimension yielded "unphysical" solutions [12], [16], [15]. Specifically, like charges appeared capable of colliding in certain circumstances while unlike charges could not collide. Alternative forms for the radiation reaction terms due to Page [17] and others (cf. a survey article of Erber [13]) were known to avoid this difficulty.

C. For the two-body problem with both retarded and advanced interactions, Schild [18] had exhibited specific circular orbit solutions. Then Andersen and von Baeyer [1], [2] had obtained solutions numerically for both one- and two-dimensional motion. No uniqueness criteria were known for this problem. However related results had been obtained for a "backwards" problem under model A [6], [20], [14], [7].

Under the present grant significant progress was made on the mathematical model C involving both retarded and advanced interactions. Consider the special case of two electrons moving symmetrically on the x-axis--one at $x(t) > 0$ and the other at $-x(t)$. Then the equations of motion become

$$x' = cv \qquad (\text{where} \quad |v| < 1), \tag{1}$$

$$\frac{v'}{(1-v^2)^{3/2}} = \frac{k}{r^2}\,\frac{1 - v(t-r)}{1 + v(t-r)} + \frac{k}{q^2}\,\frac{1 + v(t+q)}{1 - v(t+q)}\,, \tag{2}$$

$$cr = x + x(t-r) \quad \text{and} \quad cq = x + x(t+q), \tag{3}$$

where $c > 0$ is the speed of light and $k > 0$ is a constant proportional to the square of the charge on an electron divided by its rest mass. [For brevity the values of the unknowns at time t, namely, $x(t)$, $v(t)$, $r(t)$, and $q(t)$, are written simply as x, v, r, and q.] Thus, each particle is assumed to be under the influence of both the past and future performance of the other.

Equations (1), (2), and (3) are considered for $-\infty < t < \infty$ together with the special "initial" data

$$x(0) = x_0 > 0, \quad v(0) = 0. \tag{4}$$

From the fact that $k > 0$, one sees that x_0 is the least possible value of $x(t)$.

As unnatural as this model may seem, it was shown that a unique solution is determined for all time provided that the minimum separation of the two charges, $2x_0$, is at least 4600 classical electron radii [10].

Further work done under this grant has provided an elementary method for studying the asymptotic behavior of solutions of certain linear delay differential equations with constant coefficients and constant delays [9]. This should have application in the study of control systems with time lags and in various other applied problems. It also has relevance for a certain one-body problem of electrodynamics with radiation reaction terms of the type proposed by Page [17]. In its simplest form with

no external forces the equation of motion looks like

$$v'(t) = b[-v(t) + v(t-\tau)], \tag{5}$$

where b and τ are positive constants and $v(t)$ is the velocity of the particle in one dimension. For this equation the method described in [9] easily shows that every solution is bounded as $t \to \infty$. However if, as has sometimes been suggested, one replaces $v(t-\tau)$ by the first few terms of a "Taylor's expansion", say $v(t) - \tau v'(t) + \frac{1}{2}\tau^2 v''(t)$, then one obtains the unstable ordinary differential equation

$$\tfrac{1}{2}b\tau^2 v''(t) = (1 + b\tau)v'(t). \tag{6}$$

This suggests that the particle will probably accelerate indefinitely even though no external fields are present--an example of the unphysical "runaway" behavior which has long plagued the Dirac form of radiation reaction.

In joint work with H. A. Levine the method of reference [9] was applied to the partial differential "telegraph equation"

$$u_{tt} - c^2 u_{xx} + bu_t + ku = 0 \quad \text{for} \quad t \geq 0, \quad 0 \leq x \leq a. \tag{7}$$

Here $c > 0$, $b > 0$, and $k \geq 0$ are constants, and equation (7) is considered together with the homogeneous boundary conditions

$$u(t,0) = 0 \quad \text{and} \quad u(t,a) = 0 \quad \text{for} \quad t \geq 0. \tag{8}$$

One finds that any solution of Eqs. (7) and (8) tends to zero at a predictable exponential rate as $t \to \infty$. The argument involves choosing a positive $\delta < \mathrm{Re}\,[b - \sqrt{b^2-4k}\,]/2$ and then considering the "Lyapunov functional"

$$V(t) \equiv \int_0^a \tfrac{1}{2}[v_t^2 + c^2 v_x^2 + (k-b\delta+\delta^2)v^2]\, dx,$$

where $v(t,x) \equiv e^{\delta t}u(t,x)$.

The research program summarized here has been continuing, under AFOSR Contract F 49620-79-C-0129, toward a study of the n-body problem for charged particles. The simplest case--namely n charged particles moving on the x-axis under the influence of their mutual retarded interactions only--had defied even "local" solution. The difficulty arose because a resulting system of ordinary differential equations apparently failed to satisfy any known uniqueness criteria. The system has the form

$$\frac{v'}{(1-v^2)^{3/2}} = \sum_{i=2}^{n} \frac{K_i}{r_i^2} \phi_i(t-r_i),$$
$$r_i' = f_i(v, \phi_i(t-r_i)) \qquad i = 2, \ldots, n \tag{9}$$

where v is the velocity of a particle under consideration and $r_2, \ldots, r_n$ are the retardations for the fields arriving at this particle from the others. Each K_i is a constant and each f_i is a known locally Lipschitzian function with $f_i < 1$. Each ϕ_i is an absolutely continuous function related to the past history of the velocity of one of the other particles (particle i). Initial conditions would be

$$v(0) = v_0 \in (-1, 1) \quad \text{and} \quad r_i(0) = r_{0i} > 0 \quad \text{for} \quad i = 2,\ldots,n. \tag{10}$$

Local existence and uniqueness of the solution of Eqs. (9) and (10) would follow at once if the functions ϕ_i were locally Lipschitzian. But such smoothness seems to be an unreasonable assumption in the electrodynamics problem. Current work with M. J. Norris of Sandia Laboratories appears to be resolving this difficulty. Details should be available when the report for the current contract is prepared.

REFERENCES

[1] C. M. Anderson and H. C. von Baeyer, Almost circular orbits in classical action-at-a-distance electrodynamics, Phys. Rev. D 5 (1972) 802-813.

[2] _____, Solutions of the two-body problem in classical action-at-a-distance electrodynamics: straight-line motion, Phys. Rev. D 5 (1972) 2470-2476.

[3] P. A. M. Dirac, Classical theory of radiating electrons, Proc. Roy. Soc. London Ser. A 167 (1938) 148-169.

[4] R. D. Driver, A two-body problem of classical electrodynamics: the one-dimensional case, Ann. Physics 21 (1963) 122-142.

[5] _____, A functional-differential system of neutral type arising in a two-body problem of classical electrodynamics. Internat. Sympos. Nonlinear Differential Equations and Nonlinear Mechanics, pp. 474-484, Academic Press, New York, 1963.

[6] _____, A "backwards" two-body problem of classical relativistic electrodynamics, Phys. Rev. 178 (1969) 2051-2057.

[7] _____, Point data problems for functional differential equations, Dynamical Systems, An International Symposium, vol. 2, 115-121, Academic Press, New York, 1976.

[8] _____, Topologies for equations of neutral type and classical electrodynamics, Differencial'nye Uravnenija s Otklonjajuščimsja Argumentom, Kiev "Naukova Dumka" 1977, 113-127. Also as Tech. Rept. No. 60, Dept. of Math., Univ. of Rhode Island, Kingston, 1975.

[9] _____, Exponential decay in some linear delay differential equations, Amer. Math. Monthly 85 (1978) 757-760.

[10] _____, Can the future influence the present? Phys. Rev. D 19 (1979) 1098-1107.

[11] R. D. Driver and M. J. Norris, Note on uniqueness for a one-dimensional two-body problem of classical electrodynamics, Ann. Physics 42 (1967) 347-351.

[12] C. J. Eliezer, The hydrogen atom and the classical theory of radiation, Proc. Cambridge Phil. Soc. 39 (1943) 173-180.

[13] T. Erber, The classical theories of radiation reaction, Fortschr. Physik 9 (1961) 343-392.

[14] D. K. Hsing, Existence and uniqueness for the one-dimensional backwards two-body problem of electrodynamics, Phys. Rev. D 16 (1977) 974-982.

[15] D. K. Hsing and R. D. Driver, Radiation reaction in electrodynamics, in Dynamical Systems, Proceedings of a University of Florida International Symposium, pp. 427-430, Academic Press, New York, 1977.

[16] J. Huschilt and W. E. Baylis, Numerical solutions to two-body problems in classical electrodynamics: Head-on collisions with retarded fields and radiation reaction. I. Repulsive case, II. Attractive case, Phys. Rev. D 13 (1976) 3256-3268.

[17] L. Page, Is a moving mass retarded by the reaction of its own radiation?, Phys. Rev. 11 (1918) 376-400.

[18] A. Schild, Electromagnetic two-body problem, Phys. Rev. 131 (1963) 2762-2766.

[19] S. P. Travis, A one-dimensional two-body problem of classical electrodynamics, SIAM J. Appl. Math. 28 (1975) 611-632.

[20] V. I. Zhdanov, On the one-dimensional symmetric two-body problem of classical electrodynamics, Internat. J. Theoret. Phys. 15 (1976) 157-167.

PUBLICATIONS

Exponential decay in some linear delay differential equations, Amer. Math. Monthly 85 (1978) 757-760.

Can the future influence the present?, Phys. Rev. D 19 (1979) 1098-1107.

In preparation with M. J. Norris: A uniqueness theorem for ordinary differential equations applicable to an n-body problem of classical electrodynamics.

MEETINGS ATTENDED

Workshop on Classical Particle Electrodynamics, International Centre for Theoretical Physics, Trieste, Italy, 11-15 July 1977. I presented invited lectures describing the status of the two-body problem and the related functional differential equations.

American Mathematical Society 82nd Summer Meeting, Brown Univ., Providence, R.I. 9-12 August 1978. I presented an invited paper at a special session of functional differential equations: "Can the future influence the present?"

Functional Differential Eyuations and Integral Equations Conference, University of West Virginia, Morgantown, W. Va., 17-20 June 1979.